Robust Optimization for Facility Location and Transportation Design under Demand Uncertainty in Relief Distribution

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Abstract: Humanitarian logistics have been indicated as significant issues in several terms of natural disaster operations and management. Indeed, a post disaster usually meets with a fluctuation of demand as known as the demand uncertainty. Therefore, we propose a few distinct network designs both single and double hierarchies of facility sites to find a most efficient network for a relief distribution under the demand uncertainty. An objective is to search the facility locations and optimize the transportation amount that can be achieved at a minimum total delivery cost which includes transportation cost, facility cost and transshipment cost. The demand uncertainty is handled by Robust Optimization (RO) which has the capability to operate under lack of full information on the nature of uncertainty and increasing the popularity. The case study is applied and evaluated from the previous severe disasters 2011 Tohoku earthquake. We focus on the most affected area which is Miyagi prefecture.

Keywords: Robust Optimization, Demand Uncertainty, Facility Location, Relief Distribution

1. INTRODUCTION

In general, there are three steps in term of humanitarian logistics which are Pre-disaster humanitarian logistics, During-disaster humanitarian logistics and Post-disaster humanitarian logistics. The Pre-disaster humanitarian logistics include mitigation and preparedness. Next, the During-disaster humanitarian logistics are concentrated with the response. Finally, the Post-disaster humanitarian logistics are consisted of response and recovery. Logistics is mainly supported in two ways which are a victim rescuing and an essential provider to survive them. There are sub three problems in logistics activities; location, routing and location-routing which are realized with a cost efficiency, a quick response, a satisfied demand and an environment issue. Moreover, the efficiency of planning and coordinating of logistic activities are necessary to treat them.

The location problem is one of the most important aspects in logistic activities. Some researchers have been done about the appropriate location of medical centers where the evacuees can be quickly accessed. Not only the medical centers but also the location of shelters is conducted. Details of these researches are shown next part of the literature reviews. This study intends to design the depot locations by considering the cost efficiency and also
the satisfaction with the demand. Mixed integer programming is popularly applied to solve the facility location problem. A general concept is the determination of two types of variables: one is binary \((0, 1)\) variables and the other is integer variables. Mostly, the selected location variables are defined as binary variables, \(0\) is assigned to those locations which are not selected and \(1\) is assigned that those locations are selected to open as facility location. Another type of variables is integer variables. These variables are supported to obtain the transportation amount of each link that the facility locations are opened. The first general category is to minimize the total cost of delivery or distance minimization with satisfaction for all constraint. The second common category is oriented for demand objectives which consist of demand coverage and demand assignment. The remaining about ten percent of all articles belong in either of last two objective categories. One of these is profit maximization while another one is objectives that address environmental concerns.

The post disaster logistics functions are defined for two significant issues as proving essentials to survived victims and recuing the victims. This study is considered the vital item distributions to relieve the large number of survived victims. A bottle of water is considered to be a requisite item for preliminary succor. This model is to design principally the distribution network with multi-layer of facility locations by using the multi-source Capacitated Facility Location Problem (CFLP), or sometime is called the Capacitated Concentrator Location Problem (CCLP). The model is designed for single and double layers of depots to make the model more realistic and satisfied with the demand. The number of required facilities and the locations of them are the two main questions in the facility location problem. Then, the assigned link flows of every facility location are designed. The distribution networks consist of a number of suppliers at fixed locations, a number of central depots and depots sites in unknown locations (need to select from a set of potential sites), and finally a number of shelters or demand zones in fixed locations. The model of facility location and transportation design is examined to obtain the minimized total cost in the initial stage demand. There are three costs for distribution. They are travel cost, transshipment cost and opening depot cost. The initial stage is calculated with the determined demand. Then, this model is solved by mixed-integer programming, which offers the appropriate depot locations, total delivery cost and transportation amount. There are three assumptions for facility location deterministic model. The first is proposed that the set of potential central depot site is designated as the only single hierarchy, depots. The second and the third are assumed in that the set of potential central depot site is determined for double hierarchies, central depots and depots. However they are dispatched by distinct truck operation.

As we know that there are enormous impacts as both a humanitarian crisis and a massive economic aftermath of the 2011 Tohoku earthquake and tsunami. Also as mentioned before, the disaster logistic activities for both commodity dispatch and evacuations become a more serious awareness with many issues such as saving life of refugees, maximum evacuee movement, efficiency cost and so on. Their functions are to operate the rescuing and amelioration of huge amount of victims. The Tohoku’s earthquake is the greatest earthquakes in the east part of Japan. Many cities are damaged along the coast areas of Tohoku region. Moreover, the areas around Fukushima Daiichi Nuclear Power Plant are warned to be evacuation zones because of affecting from the tsunami. When considering the large amount of evacuees, there are approximately more than 340,000 people had been displaced from their accommodations to shelters. The estimation of economic losses from some analysts is evaluated as amount of 10 trillion yen for both immediate problems with industrial production suspended in many factories, and the longer term issue of the cost of rebuilding. However, the Japanese Government had estimated that this cost is much higher than the cost of just the direct material damage could exceed 25 trillion yen. Moreover, the several costs are generated
to recover the situations during disaster and post disaster period, for example reconstruction cost, rescue cost, logistics cost and etc. The logistic cost is approximately 80 percent from overall of operation responding cost. Therefore, the cost efficiency should be one of many aspects that must be considered. By this reason, this study would like to play on the logistic cost efficiency. An improved supply distribution cost can reduce the expenditure of the whole of operation cost during the amelioration period. Even the total delivery cost minimization is not only one to consider in humanitarian logistics however it is a good criterion to compare the results of distinct network systems.

Furthermore, the real situations usually meet with the fluctuation of demand known as demand uncertainty. Therefore, this study also stresses the importance of uncertainty of parameters, here is the demand uncertainty. In fact, the post disaster circumstances usually face with the fluctuation of number of evacuees and un-precisely prediction. The methodology to handle with this demand fluctuation is robust optimization. This study aims to handle the facility location problems or mixed integer problems with the uncertainty of demand. The models that illustrate for uncertainty parameters are known as robust optimization model which are opposite with deterministic models. Previously, there are various solution approaches to tackle with the uncertainty of parameters. Here is the list of some techniques which have been used in the previous, sensitivity analysis, stochastic programming, chance constraint programming, fuzzy set modeling, robust optimization, stochastic decision processes based on Markov processes and global optimization. More details are discussed in the literature reviews.

This study considers Robust Optimization (RO) which is provided by AIMMS software and more recently applied to handle under uncertainty of the parameters in the models. Robust Optimization designs to meet some major challenges associated with uncertainty-affected optimization problems as follows; to operate under lack of full information on the nature of uncertainty, to model the problem in a form that can be solved efficiently and to provide guarantees about the performance of the solutions. Robust Optimization is an uncertainty modeling approach suitable for a situation where the uncertainty ranges are known and not necessarily the distribution. Typically some inputs take an uncertain value anywhere between a fixed minimum and a maximum. This demand uncertainty can present how the worst case is when we consider the fluctuation of the demand. Robust Optimization is very suitable for many problems as only simple inputs are required from the user about the data uncertainty because there are no scenarios or distribution functions need to be defined. In contrast with Stochastic Programming that can result in large models when considering many scenarios. However, these many scenarios should make it important to limit the number of considered scenarios, but therefore also affect that the results are less robust. The advantage of Robust Optimization models is that they grow only slightly when uncertainty is added. As the result, the model can be solved efficiently.

Not only a single deterministic demand model is investigated but also multi demand scenarios are examined. This method is needed to analyze the sensitivity of three different network structures. We determine the demand for five scenarios which deviate from the historical data in both sides optimistic and pessimistic. These five demand scenarios are separated as less than actual demand by 20 percent (S1) and 10 percent (S2), actual demand (S3), and more than actual demand by 10 percent (S3) and 20 percent (S4) respectively.

The major compositions of this study for both deterministic demand and uncertainty demand are summarized as follows:

1) To search the appropriate locations of depots to distribution the relief items in Miyagi prefectures.
2) To optimize the transportation amount for each link of suppliers, facility depots
3) To minimize the total delivery cost which includes the transportation cost, transshipment cost and fixed cost of opening depots.

4) To compare the transportation cost and their behavior.

2. LITERATURE REVIEW

There were various aspects that have some dealings with this study. Thus, this section intends to review some relation of the research which can be separated into three main aspects as the ordinary location problems, the uncertainty parameter problems and the emergency relief distribution. Since before 1990 there were many existence researches of facility location decision problem. Thereby, Current et.al (1990) reviewed such kinds of this problem that play on multi-objective analysis. They introduced that the facility location decision models were attracted from several researchers in many fields since they were pioneered by Weber (1929). The models were appeared from the academic fields of economics, engineering, geography, mathematics, operation research, planning and regional science. Moreover, they categorized the objectives of the study in fours classification which are cost minimization; demand oriented, profit maximization and environmental concern.

In the point of researches, incorporating uncertainty parameters into the planning network design problem and the optimization for robust solutions have become popular and increasingly important. A robust approach to solve linear optimization problems with uncertain data was proposed in the early 1970s and has recently been extensively studied and extended and a large amount of researches are studied in the theory of robust optimization. Soyster (1970) proposed a linear model to construct solution that is feasible for all data that belong in a convex set. Kouvelis and Yu (1997) discussed the robust discrete optimization and its applications. Ben-Tal and Nemirovski (2000) studied the problem that the robust solution of linear programming problems contaminated with uncertain data. Josef (2004) gave an overview on the state-of-the-art and recent advances in mixed integer optimization to solve planning and design problems in the process in industry. They performed modeling for optimization both deterministic and under uncertainty. Stochastic programming for continuous LP problems is now part of the most of the optimization packages, and there is encouraging progress in the field of stochastic MILP and robust MILP. Bertsimas and Brown (2009) proposed a methodology for constructing uncertainty sets for robust liner optimization based on decision maker risk preferences. Ben-Tal, Bertsimas and Brown (2010) proposed a soft robust model for optimization under ambiguity. Song and Luo (2010) developed a robust portfolio selection model under ellipsoidal uncertainty. Many fields of the academic study had discussed uncertainty parameter handling with robust optimization approaches, for an instance, a design and operations of chemical processes, an electrical capacity system, supply chain networks and transportation planning design. The robust model is useful to help the planner to identify trade-offs between the inability to recover fully costs for excess link flow, and the need to manage transportation resource such as trucks, drivers and etc. to satisfy with the demand.

Thomas and Mizushima (2005) defined the humanitarian logistics that “It is the process of planning, implementing and controlling the efficient, cost-effective flow of and storage of goods and materials as well as related information, from point of origin to point of consumption for the purpose of meeting the end beneficiary’s requirements.” Van Wassenhove (2006) proposed that “the humanitarians logistics consists of the processes and systems involved in mobilizing people, resources, skills and knowledge to help vulnerable people
affected by disaster.” Nagurney et al. (2012) presented that “disaster relief and associated cost are approximately 80 percent logistics. Therefore, more transparent, efficient, and effective logistics operations and supply chain management in disaster cannot save only lives but also enables better preparedness for natural as well as man-made disaster.” These support that the logistics operation improvements are interested in the present. Lin et al. (2012) focused on logistics efficiency improvement. They said that the prioritized items for delivery and an extensive time period are the importance aspects of humanitarian logistics. They would like to present the location of temporary depots around the disaster-effected area between the long travel distances of demand points and the central depots. The two-phase heuristic approach was presented. The best solution from Phase I, which was located the temporary depots and allocate covered demand area was assigned in Phase II to achieve the best logistics performance. Wisetjindawat et al. (2012) studied on the vehicle routing model for distribution of emergency goods to victims. The time for responding was considered to develop the relief supply operation model in Aichi prefecture’s plans for the locations of hubs. This study is solved by vehicle routing problem which was restricted by the maximum load of truck and working time of drivers.

3. MATHEMATICAL MODEL

This section discusses the mathematical model both objective function and constraint function. The location facility model is based on the mixed-integer linear programming that the facility needs to be selected by given the best solution of objective value. This section is separated for two main aspects. First of all, the ordinary location facility model is performed with deterministic of demand parameters. The basic concepts and general ideas of facility location problem are regarded in this section. Afterwards, the facility location model with demand uncertainty is generated by using robust optimization approach. The methodology frameworks and notions of robust optimization are explained in this section.

Indices

\[ M \]: Set of the supplier nodes \((i)\)

\[ N \]: Set of the candidate central depots \((j)\)

\[ L \]: Set of the candidate depots \((k)\)

\[ P \]: Set of the demand nodes or shelters \((l)\)

Notations

\[ x_{ij} \]: The amount of goods from suppliers \((i)\) to central depots \((j)\)

\[ y_{jk} \]: The amount of goods from central depots \((j)\) to depots \((k)\)

\[ z_{kl} \]: The amount of goods from depots \((k)\) to demand areas \((l)\)

\[ D_Y_j \]: Demand at the candidate central depots \((j=1,2,3…N)\)

\[ C_Y_j \]: Capacity at the candidate central depots \((j=1,2,3…N)\)

\[ D_Z_k \]: Demand at the candidate depots \((k=1,2,3…L)\)

\[ C_Z_k \]: Capacity at the candidate depots \((k=1,2,3…L)\)

Sets of parameters

\[ S_i \]: Amount of items at the supply nodes \((i=1,2,3…M)\)

\[ D_l \]: Demand at the affected area nodes \((l=1,2,3…P)\)

\[ c_{ij} \]: The travel cost between \(i\) and \(j\)

\[ c_{jk} \]: The travel cost between \(j\) and \(k\)
The travel cost between \( k \) and \( l \)

The opening depot cost at \( j \)

The opening depot cost at \( k \)

The transshipment cost at \( j \)

The transshipment cost at \( k \)

The capacity of truck between \( i \) to \( j \), \( j \) to \( k \) and \( k \) to \( l \)

The working time of drivers between \( i \) to \( j \), \( j \) to \( k \) and \( k \) to \( l \)

### 3.1 Objective Function

The objective formulation is determined to minimize the total delivery cost. The first three parts of objective function are generated for the transportation cost which depends on the amount of transportation. The latter two parts are performed to calculate the opening facility cost of layer-2 and layer-3 or as known as central depots and depots. The last two parts are calculated for loading/unloading cost at each facility location.

\[
\begin{align*}
\text{min} & \left\{ \sum_{i=1}^{N} \sum_{l=1}^{M} c_{ij}x_{ij} + \sum_{k=1}^{L} \sum_{j=1}^{N} c_{jk}y_{jk} + \sum_{l=1}^{P} \sum_{k=1}^{L} c_{kl}z_{kl} + \sum_{j=1}^{N} f_{j}y_{j} + \sum_{k=1}^{L} f_{k}z_{k} \right. \\
& \left. + \sum_{j=1}^{N} t_{j}y_{j} + \sum_{k=1}^{L} t_{k}z_{k} \right\} 
\end{align*}
\]

\( (1) \)

Decision variables

\[
Y_{j} = \begin{cases} 
1, & \text{if central depots is located at } j \\
0, & \text{otherwise} 
\end{cases} \quad \text{for } j \in N 
\]

\( (2) \)

\[
Z_{k} = \begin{cases} 
1, & \text{if depots is located at } k \\
0, & \text{otherwise} 
\end{cases} \quad \text{for } k \in M 
\]

\( (3) \)

Subject to

\[
\sum_{j=1}^{N} x_{ij} \leq S_{i} \quad \text{for } i \in M 
\]

\( (4) \)

\[
\sum_{i=1}^{N} x_{ij} \leq DY_{j} \quad \text{for } j \in N 
\]

\( (5) \)

\[
\sum_{k=1}^{L} y_{jk} \leq CY_{j} \quad \text{for } j \in N 
\]

\( (6) \)

\[
\sum_{j=1}^{N} z_{jk} \leq DZ_{k} \quad \text{for } k \in M 
\]

\( (7) \)
\[
\sum_{l=1}^{P} z_{kl} \leq CZ_k \tag{8}
\]
\[
\sum_{k=1}^{L} z_{kl} \leq D_l \tag{9}
\]
\[
v_{ij}, v_{jk}, v_{kl} \leq \text{maximum truck volume} \tag{10}
\]
\[
w_{ij}, w_{jk}, w_{kl} \leq \text{maximum working time of driver} \tag{11}
\]
\[
x_{ij}, y_{jk}, z_{kl} \geq 0 \tag{12}
\]
\[
Y_j, Z_k \in \{0,1\} \text{ for all } j \text{ and } k \tag{13}
\]

The first constraint, Equation (4) is guaranteed that the total amount flow from supplier locations \( i \) to central depots \( j \) is not over than the amount of serving goods at suppliers \( i \). The second constraint, Equation (5) is restricted for the summation of link flow from \( i \) to \( j \) does not exceed than the capacity of opening the central depots \( j \). The third constraint, Equation (6) is limited that the total amount of link flows from \( j \) to \( k \) not exceeding than the total availability of goods at opening central depots \( j \). The fourth constraint, Equation (7) is constructed to limit that the summation amount of link flow from \( j \) to \( k \) must not be over than the capacity of next network configuration or depots \( k \). The fifth constraint, Equation (8) is ensured that the total amount from depots \( k \) to demand \( l \) is not over than the capacity \( C \) at depots \( k \). The sixth constraint, Equation (9) is confirmed that the total amount serving from depots \( k \) is satisfied with the demand \( l \). The seventh constraint, Equation (10) is determined to prohibit that the amount of a commodity cannot exceed the maximum truck volume restriction. The eighth constraint, Equation (11) is restricted for the total driving hours of driver which are not over than the maximum working time. The ninth constraint, Equation (12) is confirmed that each link flow from site \( i \) to \( j \), \( j \) to \( k \) and \( k \) to \( l \) need to define with some amount of goods. Finally, Equation (13) is generated to specify that the both decision variables \( Y_j \) and \( Z_k \) are binary variable 0 and 1, 1 is represented, if the facility is located at site \( j \) and \( k \) and 0 is otherwise.

### 3.2 Mathematical with Robust Formulation by using Robust Counterpart

This study focuses on the multi-source and multi-layer of facility location problem with uncertainty demand by considering the ellipsoidal uncertainty set in robust optimization approach. There are some researchers, Ben-Tal and Nemirovski and El Ghaoui et al. who consider ellipsoidal uncertainty set. Whenever the uncertainty set of a mixed-integer robust problem is an ellipsoidal, and then the robust counterpart can be reformulated as a mixed-integer second-order cone program (SOCP).

This study focuses on the demand uncertainty parameter which deviates from the estimated or nominal value of the uncertain parameter. The demand is defined as parameter \( D \) and \( (\bar{D}) \) is the demand that deviate from normal values. The demand uncertainty is expanded followed by the region of the ellipsoidal uncertainty set. Then, the robust counter part of mixed-integer programming becomes mixed-integer second-order cone program (SOCP).

\[
\text{Ellipsoidal} : \quad U = \{ D \in \mathbb{R}^d : (D - \bar{D})^t S^{-1} (D - \bar{D}) \leq \rho^2 \} \tag{14}
\]
\[
\min \left\{ \sum_{j=1}^{N} \sum_{i=1}^{M} c_{ij}x_{ij} + \sum_{k=1}^{L} \sum_{j=1}^{N} c_{jk}y_{jk} + \sum_{l=1}^{P} \sum_{k=1}^{L} c_{kl}z_{kl} + \sum_{j=1}^{N} f_{j}y_{j} + \sum_{k=1}^{L} f_{k}z_{k} \right. \\
\left. + \sum_{j=1}^{N} t_{j}y_{j} + \sum_{k=1}^{L} t_{k}z_{k} \right\} \\
\sum_{k=1}^{L} z_{kl} \leq D_{l}
\]

(15)

(16)

4. A CASE STUDY AND PROBLEM ASSUMPTIONS

4.1 Study Area

First of all, this paragraph is introduced some general information that how the severe of disaster and study area are related. As well known in the several name are the 2011 earthquake off the Pacific coast of Tohoku, the Great East Japan Earthquake and also the 2011 Tohoku earthquake, this earthquake occurred undersea with mega thrust earthquake off the coast of Japan on Friday 11 March 2011. There was the epicenter approximately 70 kilometers from east of the Oshika Peninsula of Tohoku as shown in the Figure 1 below and the hypocenter at an underwater depth of approximately 30 km. The earthquake was initially reported as 7.9 magnitudes by the USGS, and it was quickly upgraded to 8.8 magnitudes, and then increased to 8.9 magnitudes and finally up to 9.0 magnitudes. The effect of this most powerful, which was 9 magnitudes is trigger of powerful tsunami waves that reached heights of up to 40.5 meters in Sendai area, travelled up to 10 km inland. Sendai was the nearest major city to the earthquake which far 130 km from the epicenter.

![Figure 1](http://commons.wikimedia.org/wiki/File:JAPAN_EARTHQUAKE_20110311.png)

![Figure 2](http://www.mapsofworld.com/japan/prefectures/miyagi.html)

The model is applied with a case study that mainly focuses on the most affected area which is Miyagi prefecture. This paragraph explains some facts which are related to the study area. These facts are generally used to specify where should be the candidate facility location.
in case of lacking with the full data. The demand areas located cover all in Miyagi prefecture. Miyagi Prefecture is located in the central part of Tohoku and connected the Pacific Ocean. There are four boundaries which are Iwate, Akita, Yamagata and Fukushima that connect at north, northwest, west and south respectively as shown in the Figure 2. Moreover, Miyagi contains Tohoku's largest city known as Sendai city which is also the capital city. There are high mountains on the west and along the northeast coast. However the central plain around Sendai is fairly large. The prefecture is separated into 13 cities, 10 districts and 22 towns and villages belong in the districts. The demand site is located for every city, towns and villages or name as shelters which the number of site depends on the amount of evacuees.

4.2 Problem and Data Assumption

This section is an explanation for the assumptions of the problem and data that are necessary to apply in the model. The explanation is stated from the largest scope of the distribution network design which is the structure of the model. Then, the assumptions of each network configuration such as supply locations, facility depot locations, demand locations, amount of demand and circumstance are mentioned.

4.2.1 Model structure

The problem is designed for three different network frames. The distinct networks are categorized by the network configurations and the dispatched truck sizes. Two types of the network configurations are single hierarchy and double hierarchies of facility site candidates, defined as a central relief depot and a relief depot. We attempt to evaluate the model by different network configurations. Hence, the problem is imposed that there are three network configurations with two echelons and four network configurations with three echelons. The first network configuration is the locations where the serviceable supports known as suppliers. The second network configuration is the central relief depot in case of two hierarchies. The third configuration is the relief depots for double hierarchies and the relief depot in case of a single hierarchy. These second and third network locations are unknown and need to be defined with the most efficiency. Finally, a possible area that was attacked by the natural disaster is called an affected area which can be defined known locations as demands. The transportation truck sizes are a large truck and a small truck which are 10-ton trucks and 4-ton trucks. The specification of the networks is described below.

Network 1: This network is determined with the three network configurations which include suppliers, relief depots and shelter demands. The relief depot candidate sites are located inside the affected areas. The relief items are dispatched from suppliers to relief depots by using 10-ton trucks. Then, the 4-ton trucks are used for portage the relief items from relief depots to shelter demands.
Network 2: This network is determined with the four network configurations which include suppliers, central relief depots, relief depots and shelter demands. The central relief depot candidates are supposed to locate inside the affected areas. The 10-ton trucks are proposed to transport the relief items from supplies to central relief depots and central relief depots to relief depots. Then, the relief items are carried from relief depots to shelter demands by using 4-ton trucks.

Network 3: This network is duplicate structure with the Network 2 in term of the number of network configuration and their location. However, there is the difference in term of the truck size. The 10-ton trucks are assumed to deliver the relief items from supplies to central relief depots. Then, 4-ton trucks are assigned to deliver from central relief depots to relief depots and from relief depots to shelter demands respectively.

4.2.2 The number of evacuees

This study uses the number of evacuees in Miyagi prefecture to represent the demand. This amount is available in the Miyagi Prefectural Government Report. They have published the evacuation situation and circumstances of the Great East Japan Earthquake. The number of evacuees is used for estimation of the amount of relief items. The locations and the number of shelters are also illustrated. These data can locate the location shelter nodes. The data is shown that the evacuees are rather huge in the flat area and a capital city. There is the biggest demand in Sendai which is approximately 70,000. Thus, under the assumption that Sendai is divided for five of demand sites which show in Figure 6. By the way, even the amount of victims in Ishinomaki is also high. However, the demand site in Ishinomaki is not divided by
the reason that the flat area is quite small. So there is no significance in term of distance. On the other hand, there are fairly less amount of evacuees on the top site and below site are located near Sendai city. The lowest evacuees are in Marumori town where connected under Sendai, which are presented at five victims. Exempt these mentions before the amount of evacuees are quite small when comparing with Sendai and Ishinomaki. These amounts are some hundreds of people for each shelter.

![Figure 6. The number of evacuee clustering by zone](image.png)

### 4.2.3 Identify the supplier locations

This section would like to specify the facility locations of network configurations for relief distribution. These locations are represented in latitude and longitude coordinates there are assumptions that locations are easily accessible by the expressway or main road. These locations are also group for some site where no significance is in term of distance.

First of all, the locations of suppliers are determined. The Miyagi Prefectural Government Report presents that there are 29 sites cover all in Japan where are available for serving goods to the affected area during post disaster. However, these 29 sites are grouped into nine sites of suppliers by using the adjacent zone ideal. The coordinates of supplier locations are illustrated in the Figure 7 below.
4.2.4 Identify the candidate facility locations

Here, the locations of candidate central depots and candidate depot locations are defined. The candidate depot locations are indicated in total 11 places. These locations are specified by clustering zoning areas and demand size. Next, the assumption is that the central depots belong inside the demand areas or in Miyagi prefecture. These are assigned by covering to response the areas of the next layer or depots. This means that these central depots should response to the next layer by considering the zoning characteristics. The location of candidate central depots and depots are illustrated in the Figure 8.

4.2.5 The total delivery cost

The total delivery costs are under the assumptions that they are separated into three sections of travel cost, opening facility cost and transshipment cost. Firstly, the travel cost is estimated by the fuel consumption of a small truck and large truck, the size is 4-ton and 10-ton, which rate at 7.69 and 11.54 yen per kilometer respectively. This travel cost varies with the distance between origins and destinations. Moreover, the travel cost also concludes a driver salary under the working time limitation, purchase truck cost that divided per day and vehicle insurance cost that also divided per day. Secondly, the opening facility cost is determined for
the central depots and depots. This cost is estimated from the average price of rental building based on the business firms and it differs depending on each area. Finally, the transshipment cost is set about three time of ordinary business firm (15,000 yen per ton per day) for loading and unloading goods.

\[ Total \ delivery \ cost = (Unit \ Travel \ cost)(amount \ of \ link \ flow) + (opening \ facility \ cost) \]
\[ + (Unit \ Transshipment \ cost)(amount \ of \ loading \ and \ unloading \ at \ facility \ site) \] (17)

### 4.2.5.1 The travel Cost

Basically, the travel cost is changed by the distance and energy consumption rate. At the beginning, the energy consumption rate for 4-ton trucks and 10-ton trucks is set at 7.69 and 11.54 yen for a kilometer driving respectively. Then, the travel distance matrix is calculated from the coordinates of all network configurations. There are four distance matrixes starting from the suppliers to central depots, central depots to depots and depots to shelters. These distances are calculated by using coordinates based on the radian units followed by Equation (19). Afterwards, the travel cost is performed below which includes travel cost from length, driver salary, purchase truck cost and insurance cost. The driver cost is defined per hour of driving but must be not over than eight hours per day. The purchase truck cost and vehicle insurance cost is divided in one day unit.

\[ Radians = Decimal \ degree \times \frac{\pi}{180} \] (18)

\[ Distance = ACOS(SIN(Lat1) \times SIN(Lat2) + COS(Lat1) \times COS(Lat2) \times COS(Lon2 - Lon1)) \times 6371 \] (19)

\[ Travel \ cost = (energy \ consumption \ rate \times distance) + (driver \ salary \times working \ time) + purchase \ truck \ cost \]
\[ + \ vehicle \ insurance \ cost \] (20)

Large truck size (10-ton)
- salary driver = 1250 yen per hour or 10,000 yen per day
- purchase truck cost = 2063 yen per day
- vehicle insurance cost = 1032 yen per day

Small truck size (4-ton)
- salary driver = 1000 yen per hour or 8,000 yen per day
- purchase truck cost = 1375 yen per day
- vehicle insurance cost = 688 yen per day

### 4.2.5.2 Opening facility cost

Secondly, the opening facility cost or fixed cost includes construction cost, electricity cost and water supply cost as known in the other as a rental building cost. This value is estimated by average prices of rental storage building in each area of candidate central depots. This cost is generated by using the average rental cost per unit and the size of storage. It means that the average price of building rental is the function with the storage size as shown in the Table 1 and Table 2 below. However, during post disaster, the effect area has the several and severe
destruction including road disruptions, building collapses and so on. Then this is influenced in very difficult issue to obtain the suitable place where can be the candidate of central and depots. Moreover, during this time is also lack of necessary resources which includes the human resources, the electricity supplies, the water supplies, the fuel energy and especially the land areas. Therefore, the opening cost for the facility location, where plans to locate in effect areas are determined 10 percent higher than the original.

<table>
<thead>
<tr>
<th>No.</th>
<th>Candidate central depots</th>
<th>Average cost per m² (yen)</th>
<th>Estimated facility cost of 400 m² capacity (yen)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Kesennuma</td>
<td>14.12</td>
<td>5,646.87</td>
</tr>
<tr>
<td>2</td>
<td>Tome</td>
<td>439.56</td>
<td>175,824.18</td>
</tr>
<tr>
<td>3</td>
<td>Sendai(Kumagane)</td>
<td>2,247.55</td>
<td>899,019.81</td>
</tr>
<tr>
<td>4</td>
<td>Yamamoto</td>
<td>468.59</td>
<td>187,434.40</td>
</tr>
</tbody>
</table>

Table 2. The opening depots cost

<table>
<thead>
<tr>
<th>No.</th>
<th>Candidate Depots</th>
<th>Average cost per m² (yen)</th>
<th>Estimated facility cost of 280 m² capacity (yen)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tome and Kurihara</td>
<td>439.56</td>
<td>123,076.92</td>
</tr>
<tr>
<td>2</td>
<td>Kesennuma</td>
<td>14.12</td>
<td>3,952.81</td>
</tr>
<tr>
<td>3</td>
<td>North Ishinomaki</td>
<td>439.56</td>
<td>123,076.92</td>
</tr>
<tr>
<td>4</td>
<td>South Ishinomaki</td>
<td>439.56</td>
<td>123,076.92</td>
</tr>
<tr>
<td>5</td>
<td>South East Ishinomaki</td>
<td>439.56</td>
<td>123,076.92</td>
</tr>
<tr>
<td>6</td>
<td>Taiwa Town</td>
<td>1,396.36</td>
<td>390,981.80</td>
</tr>
<tr>
<td>7</td>
<td>North West Sendai</td>
<td>2,247.55</td>
<td>629,313.87</td>
</tr>
<tr>
<td>8</td>
<td>North East Sendai</td>
<td>2,247.55</td>
<td>629,313.87</td>
</tr>
<tr>
<td>9</td>
<td>Central Sendai</td>
<td>2,247.55</td>
<td>629,313.87</td>
</tr>
<tr>
<td>10</td>
<td>South East Sendai</td>
<td>2,247.55</td>
<td>629,313.87</td>
</tr>
<tr>
<td>11</td>
<td>South Sendai</td>
<td>2,247.55</td>
<td>629,313.87</td>
</tr>
</tbody>
</table>

4.2.5.3 Transshipment cost

Finally, the transshipment cost is approximately 15,000 yen per ton per day, triple times of the usual business, for loading or unloading of goods at the facility sites. This cost will generate when the set of facility sites are selected to operate. This cost includes salary, bonus, social insurance and pension saving of human resources. This value obtains from the firm’s Logistic Behavior Survey (PWRI) (Hosoya et al. 2003).

5. RESULTS

This section, we describe the computation results of three different systems networks and analyze their behaviors. We divide these three problems into two major sections which are deterministic demand and uncertainty demand. The demand in each network is deviated for two optimistic demand scenarios (-20 percent and -10 percent) and two pessimistic demand scenario (+10 percent and +20 percent) from actual demand. Then, we evaluate and compare the model sensitivity by those demand scenarios. We solve the problem by using AIMMS software and Gurobi solver. The deterministic demand model is solved by ordinary mixed-integer problem. On the other hand, the uncertainty demand model is solved by robust optimization. The total delivery cost and facility locations are principal outcome.
5.1 Facility Locations and Inventory

The Table 3 shows the facility sites that are selected and their inventory of deterministic demand which is solved by ordinary mixed-integer problem. The blank means that those facility sites are not chosen. The outputs of central depots in network 1 are all blank because we assign only one hierarchy in this network. There are two central depots which are located in Kesennuma and Tome for every demand scenarios and both network 2 and network 3. The main relief items are dispatched to the central depots at Tome until they are full capacity. Then, there are four depot locations at Kesennuma, North Ishinomaki, South Ishinomaki and South East Ishinomaki in case of demand scenarios 1 and scenario 2 of all networks. However, it needs one additional depot at Tome and Kurihara to satisfy with the demand scenario 3, scenario 4 and scenario 5. These central depots and depots are selected by the reasons that they provide the best solution or minimum of total delivery cost.

| Table 3. The opening facility locations and their inventory of deterministic demand |
|------------------------------|----------|----------|----------|----------|----------|
| Central depot                | Network 1 | Network 2 | Network 3 |
| S1  | S2  | S3  | S4  | S5  |
| Kesennuma | 105,406 | 168,585 | 231,743 | 294,934 | 358,104 |
| Tome | 400,000 | 400,000 | 400,000 | 400,000 | 400,000 |
| Sendai (Kumagane)            |          |          |          |          |          |
| Yamamoto                     |          |          |          |          |          |
| North East Sendai            |          |          |          |          |          |
| Central Sendai               |          |          |          |          |          |
| South East Sendai            |          |          |          |          |          |
| South Sendai                 |          |          |          |          |          |

Similarly as deterministic demand, the Table 4 shows the facility sites and their storage of uncertainty demand. The uncertainty demand model is solved by robust optimization and using ellipsoid uncertainty set. The two representative central depots as follows Kesennuma and Tome are selected to open for all demand scenarios of network 2 and network 3. However, the main relief items are shuffled delivery and depend on demand scenarios. The central depot at Kesennuma supports the major quantity of relief items in case of optimistic demand and actual demand scenarios, scenario 1, scenario 2 and scenario 3. Contrariwise, the relief items in pessimistic demand, scenario 4 and scenario 5 are mostly collected at Tome until it reaches to capacity limits.

| Table 4. The opening facility locations and their inventory of uncertainty demand |
|-----------------------------------|----------|----------|----------|----------|----------|
| Central depot                | Network 1 | Network 2 | Network 3 |
| S1  | S2  | S3  | S4  | S5  |
| Kesennuma | 80,857 | 107,785 | 86,163 | 87,436 | 143,704 |
| North Ishinomaki | 117,349 | 153,600 | 153,600 | 153,600 | 153,600 |
| South Ishinomaki | 153,600 | 153,600 | 153,600 | 153,600 | 153,600 |
| South East Ishinomaki | 153,600 | 153,600 | 153,600 | 153,600 | 153,600 |
| Taiwa Town                  |          |          |          |          |          |
| North West Sendai           |          |          |          |          |          |
| North East Sendai           |          |          |          |          |          |
| Central Sendai             |          |          |          |          |          |
| South East Sendai           |          |          |          |          |          |
| South Sendai               |          |          |          |          |          |
Comparing the deterministic demand and uncertainty demand, some opening depot locations in demand scenario 1 and scenario 2 are changes. In case network 1- scenario 2, the central depot at Tome and Kurihara of uncertainty demand is selected instead of South East Ishinomaki of deterministic demand. Likewise, scenario 1 of both network 2 and network 3, Tome and Kurihara of uncertainty demand are also selected instead of North Ishinomaki deterministic demand.

5.2 Total Delivery Cost

We demonstrate the calculation results and the objective function value comparison of three different networks for each deterministic demand scenario. From the results, we find that the network configurations and their systems are effect with the total delivery cost both deterministic demand and uncertainty demand as shown in the Figure 9 and Figure 10 respectively. It can be seen that the Network 2 and Network 3 as defined for two hierarchies of facility are obviously preferable cost performance when comparing with the Network 1 which is single hierarchy. The total delivery cost of Network 2 and Network 3 reduce by 17.96 percent and 16.78 percent respectively. The total delivery cost is mostly generated by travel which is more than about 90 percent and its rapid increase depends on the amount of transportation.
Figure 9. The calculation result of deterministic demand

When comparing Network 2 and Network 3, all demand scenarios in network 2 can be reduced by 1.19 percent, 2.79 percent, 6.06 percent, 2.49 percent and 1.71 percent respectively. These results demonstrate that not only network configurations but together with truck size operations are significant with total delivery cost function. By using 10-ton truck to deliver from suppliers to central depots and from central depots to depots can have a benefit of cost reduction. To apply model, we suggest to establish the central depots and use large truck to deliver both inbound and outbound.

Figure 10 shows the objective function value comparison and the calculation results of three different networks for each uncertainty demand scenario. We determine the uncertainty demand as ellipsoid uncertainty set then the model is solved by Robust Optimization. As same as deterministic demand scenarios, the network 2 presents the best alternative among three networks. However, the total delivery cost is slightly changed and becomes near each other when the demand increases. Therefore, this is the efficiency created by using robust optimization.
5.3 Sensitivity Analysis

Figure 11 illustrates the total delivery cost comparison between deterministic demand and uncertainty demand of three networks. It also shows the sensitivity analysis of objective function for each network. In fact, the total delivery cost of uncertainty demand is higher than deterministic demand because they consider all possible cases in the uncertainty region and attempt to search the best results that can be represented for all possible demands.

The range of sensitivity value comparison, Network 1 is wider range and higher sensitivity than the others, means that Network 1 is less robust than the other networks. The range scale of network 1 is approximately 0.1 to 9 million yen while there is approximately 3 to 5 million yen of sensitivity for Network 2 and Network 3.

The deterministic demand and uncertainty demand comparison, Network 2 and Network 3 are similar that by using robust optimization to handle the uncertainty demand illustrates more robustness than ordinary deterministic demand. In addition, the fluctuation of sensitivity between deterministic demand and uncertainty demand of Network 2 is less than Network 3. Meaning that Network 2 is robust than the other networks.
6. CONCLUSION

This study principally analyzes the multi-facility location problems under both deterministic demand and uncertainty demand issues. We diagnose the uncertainty demand by the reasons that it is quite difficult to predict the post disaster demand. We consider a whole distribution network starting from the beginning, suppliers until the end, demands. We apply the model with the case study in order to evaluate the total delivery cost during post disaster in Miyagi prefecture. This disaster is the Tohoku’s earthquake which occurred in 2011 of March. Thus, the data is collected based on the data that is reported by the Miyagi Prefectural Government Report. The objective function is to minimize the total delivery cost which includes the travel cost, the opening facility cost and the transshipment cost by selecting the facility sites and optimized transportation flow. We propose the three network structures which are the one network of single hierarchy facility and two network of two hierarchies with distinct truck size (large trucks and small trucks), to handle both demand known and unknown circumstances. We determine the region of uncertainty demand as ellipsoid uncertainty set. Therefore, this study can help the decision makers to prepare the appropriate network with robustness for relief distribution.

First of all, these three networks are designated to examine whether the different network configuration and their system are influenced by the total delivery cost. The calculation results both deterministic demand and uncertainty demand demonstrate that the network configurations are significant with total delivery cost. It can be seen clearly that the total delivery cost of network 2 and network 3 can reduce about 18 percent because the travel cost much reduces even though it requires more facility cost and transshipment cost. Therefore, we conclude that the two hierarchies of facility location show more cost efficiency than the single hierarchy. The results show that the travel cost has more significance than the opening facility cost. Moreover, the truck size operation is significant when the demand is high enough. This study found that large truck is appropriate to deliver both inbound and outbound at the central depots.

Furthermore, we also would prove that the networks are robust when the demand becomes uncertain or unknown. Here, we assume five different demand scenarios in each network based on the actual number of evacuees during post disaster as follows less than actual demand by 20 percent and 10 percent, actual demand, and more than actual demand by 10 percent and 20 percent respectively. We use these demand scenarios to investigate the model sensitivity. After solving the uncertainty demand by using robust optimization, the results prove that the structural networks affect on the model robustness. The two hierarchies of facility provide an extra robustness than the single hierarchy of facility. Moreover, the uncertainty demand model is robust than deterministic demand model.

Finally, we discuss the interrelated aspects to improve the future work as follows: (1) we have not considered the other parameters that can be possible to fluctuate during humanitarian logistics, for example the supply amount, the unit transportation cost, the opening facility cost and etc. Therefore, not only the uncertainty demand but also such kind of parameters should be considered simultaneously. (2) A new research can be improved with
more efficiency by considering the vehicle routing problem together with our facility location problem simultaneously. This model can be referred to Location-Routing problem. The model might give more interesting results because the travel cost would be reduced by using milk run.

REFERENCES


