Dynamic Journey Time Estimation in Stochastic Road Networks with Uncertainty

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Abstract: This paper investigates the dynamic journey time estimation (DJTE) problem, which can be used to estimate simultaneously the mean and standard deviation (SD) of path journey times in stochastic road networks. In this paper, a bi-level programming model is proposed to solve the DJTE problem. The upper level is to minimize the deviations of the mean and SD between the estimated and observed path journey times. The low level is a reliability-based dynamic traffic assignment model with taking account the variation of path journey times. The modified simulated annealing (SA) algorithm is adapted to solve the proposed bi-level problem. Numerical example is used to illustrate the application of the proposed DJTE model and the modified algorithm together with some insightful findings.

Keywords: Dynamic journey time; Dynamic traffic assignment; Simulated annealing algorithm.

1. INTRODUCTION

Journey time (travel time) is one of the most important measures for evaluating the performance of road network. Thus, accurate and reliable travel time information becomes increasingly important for traffic engineers. Travel time is also one of the most well-known measures for road users, helping them to make decisions on travel choice and to avoid unnecessary delay (Liu and Ma, 2009). Accurate estimation of travel times is important for improving traffic operations and identifying key bottlenecks in the traffic network (Zhan et al., 2013). Various models have been proposed to contribute to the field of journey time estimation. These models can be classified into two categories: static models for long term purposes (Lam et al., 2002; 2006; Shao et al., 2013; Zhan et al., 2013; Jenelius and Koutsopoulos, 2013; Uchida, 2014) and dynamic models for short term purposes (Dion and Rakha, 2006; Liu et al., 2007; Liu et al., 2007; Kamga et al., 2011; Sumalee et al., 2013). The static models focus on the static description of traffic flows on the network, implying that traffic flows and travel times are invariant over the duration of the peak period. The dynamic journey time estimation (DJTE) model developed in this paper belongs in the latter case which is proposed in response to the untenable assumption of static traffic flows. In this model, in contrast to the average travel times for the entire peak period estimated in the static
models, the travel times for travelers depart at different times can be separately estimated. To accurately estimate travel times within road network, these models have proposed the use of different types of traffic data collected by various technologies: electronic distance-measuring instruments, electronic license plate matching, cellular phone tracking, automatic vehicle identification (AVI), automatic vehicle location, Global Positioning System, probe vehicle, virtual probe vehicle and wireless magnetic sensors (Shao et al., 2013). In general, AVI is a proven technology for providing area-wide real-time traffic data with low operating cost. The AVI technology is suitable for annual, daily and real-time traffic monitoring (Tam and Lam, 2008). In the light of this, AVI is adopted in this paper for collecting the real-time link traffic flows and path travel times.

However, travel time estimation in traffic network is one challenging subject because the travel time is intrinsically uncertain (Zhang and Zuylen, 2013). The uncertainty of network travel times exists in both supply side (e.g. weather conditions, traffic incidents, capacity variation) and demand side (e.g. population characteristics, traffic information, and demand fluctuation) (Chen and Zhou, 2010). It is difficult for travelers to know exactly when they will arrive at their destination. Hence, evaluating only the mean of travel times may not be sufficient to capture the variability and reliability of the travel times. In view of this, this paper proposes a model for simultaneously estimating the mean and variance of path travel times to effectively identify the path travel times’ variability. It is reasonable that the mean and variance measures of travel time could theoretically and practically represent the travel time variability (Jackson and Jucker, 1981). With the help of empirical study, Jackson and Jucker (1981) concluded that the perceived travel time reliability is an important component of traveler’s route choice decision, and suggested that including the variability of travel time in the impedance function with mean travel time might improve the traffic assignment process.

In order to hedge against travel time unreliability, Hall (1983) has considered a safety margin in travel time that travelers tend to reserve in order to improve their likelihood of arriving on time. In the empirical study of Lam and Small (2001), it is reported that travelers are likely to set up a travel time safety margin to avoid late arrival. It is undoubted that the travel time and its reliability influence travelers’ route choice. Many models (Lo et al., 2006; Shao et al., 2006, 2008; Chen et al., 2012) were proposed to capture the risk-based route choice behavior using the concept of effective travel time or travel time budget, which is defined as the mean travel time plus a safety margin. With such definition, these models ensure that the probability of completing the trip within the effective travel time or travel time budget is not less than the predefined reliability threshold or confidence level $\alpha$. In view of this, the effective travel time is also adopted in this paper as travelers’ route choice criterion. In the proposed DJTE model, a dynamic traffic assignment is employed to model the travelers’ route choice behavior in dynamic networks.

Lam et al. (2002, 2006) have developed a traffic flow simulator, which based on a probit-type stochastic user equilibrium (SUE), to estimate travel times by minimizing the deviation between the observed and estimated link flows and origin-destination (OD) demands. Shao et al. (2013) proposed a journey time estimator to estimate the stochastic journey time by taking into account the reliability-based stochastic user equilibrium traffic assignment problem. Zhan et al. (2013) presented a new descriptive model for estimating the hourly average of urban link travel times using taxiOD trip data by minimizing the error between the estimated and observed path travel times. Jenelius and Koutsopoulos (2013) presented a statistical model for urban road network travel time estimation based on low frequency GPS probes vehicle data. Travel times estimated in Jenelius and Koutsopoulos (2013) are consists of the running times on links and turn delays at the traffic
signals/intersections. Uchida (2014) proposed a utility maximization framework for the simultaneously estimation of the value of travel time and the travel time reliability based on the risk-averse driver’s route choice behavior. All of the studies (Lam et al. 2002; 2006; Shao et al., 2013; Zhan et al., 2013; Jenelius and Koutsopoulos, 2013; Uchida, 2014) mentioned above fall within the class of static model. Disadvantages of such static models are that they do not capture the time-dependent characteristics of traffic flow, and they have assumed the observed link flows represent a steady-state situation that persists over a block of time (Sherali and Park, 2001).

Most of the existing dynamic travel time estimation models have made use of the mean travel times as the route choice criterion but ignore their variances. Kamga et al. (2011) presented a methodology to estimate travel time using a simulation-based dynamic traffic assignment model. Sumalee et al. (2013) proposed a framework for evaluating the distributions of stochastic dynamic link travel time and journey time as well as assessing the journey time reliability. But both mean and variance of travel time should be included as part of the impedance function in both the route choice and the modeling process (Jackson and Jucker, 1981). One advantage of the model proposed in this paper over the above dynamic models is that the mean and variance of travel time are estimated by taking account of the travel’s route choice behavior in a road network. Liu et al. (2007) estimated dynamic travel times by minimizing the summation errors of the mean and standard deviation between the observed and estimated travel times, and adopted a mixed logit formulation to compute the travelers’ route choice probability. Although using this method to capture travelers’ route choice is certainly a calculation convenience, it is not suitable for dynamic network.

Covariance relationship is another crucial information to be used in the travel time estimation process. The covariance relationships of link travel times can represent the magnitude of the dependency of travel times on two alternative links. Moreover, such dependency can provide useful information for path travel time estimation (Shao et al., 2013). Lam et al. (2002) have concluded that the covariance information have great influence on the accuracy of the travel time forecasting. Xu (2003) has conducted sensitivity tests and found that the covariance information can improve the travel time estimation, particularly for congested road network.

Similar to the previous travel time estimation problem (Shao et al., 2013), the DJTE proposed in this paper can be formulated as a bi-level problem. The upper level problem of the DJTE is a generalized least squares optimization problem which minimizes the deviations of the mean and SD between observed and estimated path travel times. The low level problem formulates travelers’ route choice behavior as a reliability-based DTA problem, which considers the covariance information of link travel times. The contributions of this paper include the followings: (1) Mean and standard deviation of path travel times are simultaneously estimated by considering travelers’ route choice. (2) Travel time and its reliability are incorporated into dynamic traffic assignment. (3) Covariance information of link travel times are taken into account in the travel time estimation process.

The remaining of this paper is organized as follows. In the next section, the notation and bi-level model formulation of the dynamic journey time estimation (DJTE) problem is firstly described. In section 3, a simulated annealing based solution algorithm is proposed to solve the bi-level problem. Numerical example of a simple small network is depicted in section 4, of which the applications of the DJTE and the efficiency of proposed solution algorithm is demonstrated. Finally, conclusions and further studies are drawn in section 5.

2. MODEL FORMULATION
2.1 Notations and Assumptions

The notations used throughout this paper are listed in Table 1 unless otherwise specified.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G = (N,A)$</td>
<td>A network composed of a set of nodes $N$, and a set of links $A$</td>
</tr>
<tr>
<td>$r$</td>
<td>Origin node</td>
</tr>
<tr>
<td>$s$</td>
<td>Destination node</td>
</tr>
<tr>
<td>$R$</td>
<td>Set of OD pairs</td>
</tr>
<tr>
<td>$P_{rs}$</td>
<td>Set of paths between OD pair $(r,s)$</td>
</tr>
<tr>
<td>$A$</td>
<td>Observed link set in which the traffic flow can be observed</td>
</tr>
<tr>
<td>$P_a$</td>
<td>Observed path set in which the travel time can be observed</td>
</tr>
<tr>
<td>$T_d$</td>
<td>Set of the departure time intervals</td>
</tr>
<tr>
<td>$T_o$</td>
<td>Set of the observed time intervals $T_o \subseteq T_d$</td>
</tr>
<tr>
<td>$C_a$</td>
<td>Capacity of link $a \in A$</td>
</tr>
<tr>
<td>$T_a'$</td>
<td>Free flow travel time of link $a \in A$</td>
</tr>
<tr>
<td>$k$</td>
<td>Subscript for the departure time intervals $k \in T_d$</td>
</tr>
<tr>
<td>$t$</td>
<td>Subscript for the observation time intervals $t \in T_o$</td>
</tr>
<tr>
<td>$x_a(k)$</td>
<td>Link flow on link $a \in A$ at the beginning of time interval $k$</td>
</tr>
<tr>
<td>$u_a(k)$</td>
<td>Link inflow rate to link $a \in A$ at the beginning of time interval $k$</td>
</tr>
<tr>
<td>$v_a(k)$</td>
<td>Link exit flow rate from link $a \in A$ at the beginning of time interval $k$</td>
</tr>
<tr>
<td>$T_a(k)$</td>
<td>Travel time of link $a \in A$ at the end of time interval $k$</td>
</tr>
<tr>
<td>$t_o(k)$</td>
<td>Mean of travel time of link $a \in A$ at the end of time interval $k$</td>
</tr>
<tr>
<td>$\sigma_a(k)$</td>
<td>SD of travel time of link $a \in A$ at the end of time interval $k$</td>
</tr>
<tr>
<td>$d^{rs}(k)$</td>
<td>Travel demand from OD pair $(r,s)$ at the beginning of time interval $k$</td>
</tr>
<tr>
<td>$f^{rs}_{p,k}(k)$</td>
<td>Flow on path $p \in P_{rs}$ at the beginning of time interval $k$</td>
</tr>
<tr>
<td>$T_{rs}^{p,k}(k)$</td>
<td>Travel time on path $p \in P_{rs}$ at the end of time interval $k$</td>
</tr>
<tr>
<td>$t_{rs}^{p,k}(k)$</td>
<td>Mean of travel time on path $p \in P_{rs}$ at the end of time interval $k$</td>
</tr>
<tr>
<td>$\sigma_{rs}^{p,k}(k)$</td>
<td>SD of travel time on path $p \in P_{rs}$ at the end of time interval $k$</td>
</tr>
<tr>
<td>$\eta_{rs}^{p,a}(k)$</td>
<td>Effective path journey time on path $p \in P_{rs}$ at the end of time interval $k$</td>
</tr>
<tr>
<td>$\pi_{rs}^{p,a}(k)$</td>
<td>Minimum effective path journey time on path $p \in P_{rs}$ at the end of time interval $k$</td>
</tr>
<tr>
<td>$\delta^{rs}_{a,k}(l)$</td>
<td>$\delta^{rs}<em>{a,k}(l) = 1$ if trips on the path $p$ of OD pair $(r,s)$ entering the network at interval $k$ and arrive link $a$ at interval $l$; Otherwise $\delta^{rs}</em>{a,k}(l) = 0$.</td>
</tr>
<tr>
<td>$\delta_{a,p}$</td>
<td>$\delta_{a,p} = 1$ if link $a$ on path $p$, otherwise $\delta_{a,p} = 0$.</td>
</tr>
<tr>
<td>$x_{a,t}(t)$</td>
<td>Estimated link flow on observed link $a \in \tilde{A}$ at the beginning of observed time interval $t$</td>
</tr>
<tr>
<td>$t_{rs}^{p,t}(t)$</td>
<td>Estimated mean travel time on path $p \in P_{rs}$ at the end of observed time interval $t$</td>
</tr>
<tr>
<td>$\sigma_{rs}^{p,t}(t)$</td>
<td>Estimated SD of travel time on path $p \in P_{rs}$ at the end of observed time interval $t$</td>
</tr>
<tr>
<td>$\tilde{x}_{a,t}(t)$</td>
<td>Observed link flow on link $a$ at the beginning of observed time interval $t$</td>
</tr>
<tr>
<td>$\bar{t}^\alpha_p(t)$</td>
<td>Observed mean of travel time on path $p \in P_{rs}$ at the end of observed time interval $t$</td>
</tr>
<tr>
<td>------------------------</td>
<td>----------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>$\hat{\sigma}^\alpha_p(t)$</td>
<td>Observed SD of travel time on path $p \in P_{rs}$ at the end of observed time interval $t$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Tolerance for the estimated link flows deviated from the observed link flows</td>
</tr>
<tr>
<td>$\lambda_{rs}$</td>
<td>Demand multiplier to the prior mean OD demand $d^r$ for $rs \in R$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Vector of $(\ldots, \lambda_{rs}, \ldots)^T$ for $rs \in R$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Non-negative multiplier between the mean and variance of the link travel time</td>
</tr>
<tr>
<td>$f$</td>
<td>Traffic pattern</td>
</tr>
</tbody>
</table>

To facilitate the presentation of the essential ideas without loss of generality, the following basic assumptions are made in this paper.

A1. It is assumed that the journey time variations are resulted from the road supply uncertainty. The observed data under demand uncertainty are excluded from the analysis in this paper. The time dependent travel demands between each OD pair are assumed to be given and deterministic. It is because the travel demand considered here is short term and within day (Corthout et al., 2010; Knoop et al., 2010; Kamga et al., 2011).

A2. The stochastic dynamic link performance function is used for the flow propagation component so as to capture the stochastic effects. Empirical results show that traffic parameters, e.g., free flow speed, capacity, etc., have large variations due to the variability in driving behavior and the characteristics of vehicles (Wang et al., 2013). Similar to Lam et al., (2008), the free flow travel time and capacity are assumed to be random parameters in the link performance function.

A3. Travelers make path choice decision based on the effective path travel time. The user equilibrium based on effective travel time or travel time budget is an extension of the classic Wardrop equilibrium, and had been adopted in static models (Lo et al., 2006; Shao et al., 2006, 2008; Chen et al., 2012). In this paper, it is further extended to dynamic networks. The covariance between link travel times are considered in calculating the effective path travel time.

A4. The variance of the link travel time is assumed to be an increasing function with respect to the mean of the link travel time.

### 2.2 Definition of Effective Path Travel Time

In this paper, we choose the following linear form for the link travel time function:

$$T_a(k) = T_a^f + \omega_a \frac{x_a(k)}{C_a}$$

where $\omega_a$ is the congestion-dependent coefficient in the link travel time function. $T_a^f$ and $C_a$ are free flow travel time and capacity, respectively. They are random variables to describe the stochastic characteristics of network supply uncertainties. Similar to the previous related studies (e.g. Lam et al., 2008), it is assumed that $T_a^f$ and $1/C_a$ are independent random variables with normal distributions. The mean of $T_a^f$ and $1/C_a$ are respectively taken as $\mu_a^T$ and $\mu_a^C$. According to Equation (1) and assumption A4, the mean and SD of the stochastic link travel time can be expressed as follows:
where $\beta$ is non-negative decision variable to be estimated by DJTE. Assume a path $p$ between OD pair $(r,s)$ is consist of the following sets of links $\{a_1,a_2,\cdots,a_m\}$ and nodes $(r,1,2,\cdots,m-1,s)$. The path travel time $T_p^{\alpha}(k)$ follows a multivariate normal distribution. The mean $(t_p^{\alpha}(k))$ and SD $(\sigma_p^{\alpha}(k))$ of the stochastic path travel time $T_p^{\alpha}(k)$ for this path $p$ can be calculated as

$$t_p^{\alpha}(k) = \sum_{a \in \text{path } p \setminus \{s\}} \sum_{l \in T_a} t_a(l)\delta_{apk}^{\alpha}(l)$$

$$\sigma_p^{\alpha}(k) = \sqrt{\left( \sum_{a \in \text{path } p \setminus \{s\}} \sum_{l \in T_a} \sigma_a(l)\delta_{apk}^{\alpha}(l) \right)^2 + 2 \sum_{l \in \text{path } p \setminus \{s\}} \text{cov}(t_a^{\alpha}(l), t_a^{\alpha}(l))}$$

where $\text{cov}(t_a^{\alpha}(l), t_a^{\alpha}(l))$ is the covariance between travel time for link $a_i$ and $a_j$ on path $p$. For instance, if the covariance between travel time for link $a_i$ and $a_j$ is positive, it indicates that the longer the travel time on link $a_i$ is correspond to the long travel time on link and $a_j$.

As mentioned above, due to the variations of path travel time, it is realistic to assume that travelers take an extra travel time budget to guarantee the on-time arrival with a desired probability while making their path choice. Similarly to the previous research (Lo et al., 2006; Chen et al., 2012), we employ the following expression to define the effective path journey time required to complete traversing path $p$ with a given probability $\alpha$.

$$\eta_p^{\alpha,\alpha}(k) = t_p^{\alpha}(k) + Z_\alpha \sigma_p^{\alpha}(k)$$

where $Z_\alpha$ is the inverse of the cumulative distribution function of standard normal distribution at $\alpha$ confidence level, which could be predetermined based on travelers’ trip purpose. For instance, if $Z_\alpha$ is taken as 1.64 in Equation (6), travelers have a 95% chance (i.e., $\alpha=95\%$) of arriving within their destination within the effective path journey time calculated in Equation (6).

To calculate the value of $\eta_p^{\alpha,\alpha}(k)$, the link travel time $T_a^{\alpha}(k)$, which can be calculated by the traffic pattern $f$ (such as link traffic flows $x_a(k)$, $\delta_{apk}^{\alpha}(l)$), and link travel time covariance $\text{cov}(t_a^{\alpha}(l), t_a^{\alpha}(l))$, should first be obtained. The traffic pattern can be obtained by solving the dynamic traffic assignment problem, which will be discussed in detail in the following section. For link travel time covariance, according to Chen et al., (2012), there is still no empirical finding on the temporal correlation between link travel times. Thus, in this paper, only the spatial covariance among link travel times is considered. For simplicity, the covariance between $t_a^{\alpha}(k_1)$ and $t_b^{\alpha}(k_2)$ is expressed as:

$$\text{cov}(t_a^{\alpha}(k_1), t_b^{\alpha}(k_2)) = \theta \exp(-\text{order}(a) - \text{order}(b))\sigma_a^{\alpha}(k_1)\sigma_b^{\alpha}(k_2)$$

where link $a$ and $b$ are on the same path $p$ and $k_1, k_2 \in T_a$; the function $\text{order}(a)$ is the ordinal number of link $a$ on the path $p$; $\theta$ has a deterministic value with $\theta > 0$ ($\theta < 0$).
indicates the link travel times are positively (negatively) correlated. For $\theta = 0$, there is no correlation between link travel times.

2.3 DTA Based on Effective Path Travel Time

If all travelers choose path based on effective path journey time, the dynamic user equilibrium condition implies that at each time interval $k$ any used path has the identical and minimum percentile path travel time (Equation 8):

$$
\eta_{rs}^{\alpha}(k) \begin{cases} 
= \pi_{rs}^{\alpha}(k), & \text{if } f_p^{rs}(k) > 0 \\
\geq \pi_{rs}^{\alpha}(k), & \text{if } f_p^{rs}(k) = 0
\end{cases} \tag{8}
$$

$$
D^{rs}(k) = \sum_{p \in P_{rs}} f_p^{rs}(k), \forall r,s,k \tag{9}
$$

$$
\nu_s(k) = \sum_{p \in P_s} \delta_{ap} f_p^{rs}(k), \forall r,s,a,p \tag{10}
$$

where Equation (9) and (10) is the path flow assignment constraints for the DTA. It is assumed that the estimated OD demand is in proportion to the prior OD demand, i.e. $D^{rs}(k) = \lambda_{rs} d^{rs}(k)$, where $d^{rs}(k)$ is given prior mean OD demand and $\lambda_{rs}$ is the demand multiplier, which is a decision variable to be estimated by the DJTE. Similar to the previous DTA problems (Huang and Lam, 2002; Yin et al., 2004; Long et al., 2011, 2013(a), 2013(b)), the flow conservation and propagation equations (Equation 11 ~ 13), definitional constraints (Equation 14) and non-negativity conditions (Equation 15 and 16) for this DTA problem is defined as follows:

$$
\sum_{a \in A(j)} u_{a,p}^{rs}(k) = \sum_{a \in A(i)} v_{a,p}^{rs}(k) \quad \forall r,s,p; \forall j \neq r,s \tag{11}
$$

$$
\frac{dx_{a,p}^{rs}(k)}{dt} = u_{a,p}^{rs}(k) - v_{a,p}^{rs}(k) \forall r,s,a,p \tag{12}
$$

$$
v_{a,p}^{rs}(k + t_{a,p}^{rs}(k)) = \frac{u_{a,p}^{rs}(k)}{1 + dt_{a,p}^{rs}(k)/dk} \quad \forall r,s,a,p \tag{13}
$$

$$
\sum_{p \in P_a} u_{a,p}^{rs}(k) = u_a(k), \sum_{p \in P_a} v_{a,p}^{rs}(k) = v_a(k), \sum_{rp} x_{a,p}^{rs}(k) = x_a(k) \quad \forall a \tag{14}
$$

$$
u_{a,p}^{rs}(k) \geq 0, v_{a,p}^{rs}(k) \geq 0, x_{a,p}^{rs}(k) \geq 0 \quad \forall r,s,a,p \tag{15}
$$

$$
f_p^{rs}(k) \geq 0, \quad \forall r,s,p,k \tag{16}
$$

According to above equations, the mean and SD of path travel time can be written as a function of $\lambda$, $\beta$, $f$ and $k$ as follows.

$$
t^{rs}_p(k) = t^{rs}_p(\lambda, \beta, f, k) \tag{17}
$$

$$
\sigma^{rs}_p(k) = \sigma^{rs}_p(\lambda, \beta, f, k) \tag{18}
$$

2.4 Upper-Level Model

The upper-level model aims to determine $\lambda$ and $\beta$. A generalized least squares function is
adopted as the objective function so that the deviations between the observed and estimated mean and SD of path travel time are minimized. Apart from the observed travel times, the observed link flows are treated as the constraints in this upper-level model. The upper-level model is formulated as follows.

$$\min_{\lambda, \beta} z_i = \gamma_1 \sum_{n \in T_e} \sum_{p \in P_e, r \in R} (t''_p (\lambda, \beta, \mathbf{f}, t) - \bar{t}'_p (t))^2 + \gamma_2 \sum_{n \in T_e} \sum_{p \in P_e, r \in R} (\sigma''_p (\lambda, \beta, \mathbf{f}, t) - \sigma''_p (t))^2$$  \hspace{1cm} (19)

subject to

$$\frac{1}{1 - \rho} \sum_{\alpha \in \mathcal{T}_e} \bar{x}_{\alpha} (t) \leq \sum_{\alpha \in \mathcal{T}_e} x_{\alpha} (t) \leq \left( 1 + \rho \right) \sum_{\alpha \in \mathcal{T}_e} \bar{x}_{\alpha} (t) \quad \forall \alpha \in \tilde{A}$$  \hspace{1cm} (20)

$$\lambda_{rs} \leq \lambda_{rs}^c \leq \lambda_{rs}^e$$  \hspace{1cm} (21)

$$\beta \geq 0$$  \hspace{1cm} (22)

Objective function Equation (19) is to minimize the weighted sum of deviation between estimated and observed mean and SD of path travel time. Constraint (20) ensures the estimated link flows are within a reasonable range of observed link flows. Constraint (21) and (22) are boundary constraints of the decision variance $\lambda$ and $\beta$, respectively.

2.5 Lower-Level Model

The discrete time version of DTA model that based on effective path journey time can be formulated as an equivalent variational inequality (VI) problem for finding a vector $\mathbf{f} \in \mathcal{F}$, such that for all $\mathbf{f} \in \mathcal{F}$ Equation (23) is satisfied.

$$\sum_{k \in T_e} \sum_{p \in P_e} \pi''_{\alpha \alpha} (k) \left[ f''_p (k) - f''_p^{**} (k) \right] \geq 0$$  \hspace{1cm} (23)

where $\mathcal{F}$ is a closed convex set defined by

$$\mathcal{F} = \{ \mathbf{f} \geq 0 : \sum_{p \in P_e} f''_p (k) = D'' (k), \forall rs \in R, k \in T_d \}$$  \hspace{1cm} (24)

Considering the upper and low level models describe in Section 2.4 and 2.5, it is clear that the lower-level problem is to determine the traffic pattern $\mathbf{f}$ with fixed $\lambda$ and $\beta$ given by the upper level. With the traffic pattern $\mathbf{f}$ from the lower level model, the upper level problem aims to find the optimal valued $\lambda$ and $\beta$.

3. SOLUTION ALGORITHM

In general the bi-level DJTE problem is solved as follows. For an initial $\lambda$ and $\beta$ in the upper level, traffic pattern $\mathbf{f}$ could be obtained through the solving of DTA in the lower level model. With the traffic pattern $\mathbf{f}$ from the lower level model and the initial value of $\lambda$ and $\beta$, the objective function of upper level could be calculated and a new set of $\lambda$ and $\beta$ could be estimated. Then the above procedure will be repeated until it is converged to the optimal solution of the bi-level model. In this section, the method to obtain the traffic pattern $\mathbf{f}$ by solving the DTA will first be presented. Then, the use of SA approach for solving the bi-level
programming model will be illustrated.

3.1 Solution algorithm for DTA

In the effective path journey time calculation process, due to the non-additive properties, link based algorithm cannot be used to solve the DTA model. Thus, the path-based algorithm that based on the Method of Successive Average (MSA) and column generation method in updating the route set is adopted.

Step 1 Initialization: Set all link flows \( x_a^u(k), u_a^v(k), v_a^w(k), \forall k \in T_d \) to zero and calculate the initial link travel time \( t_a^u(k), \forall k \in T_d \). Set the iteration counter \( n=1 \), the maximum iteration \( N \), and the convergence criterion \( \varepsilon \).

Step 2 Shortest path: Find the shortest path \( p_{rs}^{rs,a}(k) \) for each OD pair at each time interval and calculate the corresponding \( \pi_{rs,a}(k) \). If the shortest path is a new one, then update the path set \( P \).

Step 3 Flow assignment: Assign all OD demand on the shortest path based on effective path travel time, then get \( f_{rs}^{rs,a}(k) \). Calculate the new path flow

\[
 f_{sp}^{rs,a}(k) = f_{sp}^{rs,a-1}(k) + \frac{1}{n+1} \left[ f_{sp}^{rs,a}(k) - f_{sp}^{rs,a-1}(k) \right]
\]

Step 4 Network flow loading: Update all link flows \( x_a^u(k), u_a^v(k), v_a^w(k) \) and the link travel time \( t_a^u(k) \), based on the new path flow in Step 3. Then calculate effective path journey time \( \eta_{p,rs,a}(k) \) considering link travel time covariance.

Step 5 Convergence check: If

\[
 \sum_{rs} \left| \eta_{p,rs,a}(k) - \pi_{rs,a}(k) \right| f_{rs} \leq \varepsilon \quad \text{or} \quad n=N, \text{stop; otherwise,} \quad n=n+1, \text{go to Step 2.}
\]

3.2 Solution Algorithm for Bi-Level Model

In general, it is difficult to solve the bi-level programming problem, because the bi-level programming problem is a NP-hard problem. In recent years, meta-heuristic algorithms (e.g., genetic algorithm, simulated annealing, etc.) become new method to solve the combinational optimal problem. Many scholars solve the bi-level programming problem with genetic algorithm (GA) (Xu et al., 2004) and simulated annealing (SA) algorithm (Sahin and Ciric, 1998). For detailed information of SA, readers can refer to Laarhoven and Aarts (1988). In this paper, an improved SA algorithm is proposed to solve the DJTE.

3.2.1 Solution representation and neighborhood function

In this paper, real number encoding scheme is used for solution representation (i.e. \( \lambda = 1 \) represents that the demand multiplier to the prior mean OD demand \( d^a \) is 1; \( \beta = 0.5 \) represents that the multiplier between the mean and variance of the link travel time is 0.5). Under this solution representation, the following neighborhood function (Equation 25) is adopted to obtain new solutions \( \lambda' \) and \( \beta' \) from the current solutions \( \lambda \) and \( \beta \) respectively. Note that both solutions must satisfy the constraints (21) and (22).
\[ \lambda^i = \lambda^0 + \Delta \lambda, \quad \beta^i = \beta^0 + \Delta \beta \] \hspace{1cm} (25)

### 3.2.2 Fitness evaluation

From Section 2.4, it could be seen that the upper level model is a constrained optimization problem. In order to simplify the solving of the upper level problem, the constrained optimization problem will be first transformed into an unconstrained optimization problem. Since the solutions generated by neighborhood function meet the constraints (21) and (22), only constraint (20) is left over to handle. In this paper, the penalty function method is adopted for its simplicity and efficiency. Considering constraint (20), we can update the fitness function as follow:

\[
E = Z_i + M \left( | \min \left( \sum_{n \in T_a} x_a(t) - (1 - \rho) \sum_{n \in T_a} \tilde{x}_a(t), 0 \right) | \right) \\
+ M \left( | \min \left( (1 + \rho) \sum_{n \in T} \tilde{x}_a(t) - \sum_{n \in T_a} x_a(t), 0 \right) | \right)
\] \hspace{1cm} (26)

where \( M \) is a large positive constant, which can be considered as the penalty cost.

### 3.2.3 Rule to accept new solution and cooling schedule

Let \( E^0 \) and \( E^1 \) be the fitness value of the current and new solution respectively. Set \( \Delta E = E^1 - E^0 \), if \( \Delta E < 0 \), then the new solutions are accepted and considered as the current solution. Otherwise, the new solutions are accepted with the probability \( p = e^{-\Delta E/T} \), where \( T \) is annealing temperature. This setting allows the SA to accept worse solution with the probability so as to guarantee the SA can easily escape from local optimum. In order to keep the best solution, we use a memory container to reserve the temporary best solutions, and update the best solutions in time. The cooling schedule in this paper is given as

\[
T_{\text{new}} = T_{\text{old}} \times \Delta t
\] \hspace{1cm} (27)

where \( \Delta t \in (0,1) \) is temperature decrementing factor. In this cooling schedule the temperature is kept fixed at each inner loop (Step 3 ~ Step 6 in Section 3.2.5).

### 3.2.4 Convergence criteria

The convergence criterion of outer loop (Step 3 ~ Step 7 in Section 3.2.5) is that the SA stops when the current temperature is equal to the given final temperature \( T_f \).

The convergence criterion of inner loop (Step 3 ~ Step 6 in Section 3.2.5) for standard SA is to meet the given number of iterations, \( N_s \). If \( N_s \) is too small, SA will easily fall into local optimum, and, on the other hand, if \( N_s \) is too larger, the SA will spend a lot of time to obtain the optimal solution. However, to guarantee the solution quality, \( N_s \) will usually be given a relatively large number. To overcome the shortcoming of the standard SA in lengthy evaluation time, improved convergence criteria are proposed in this paper. In this paper, the inner loop stops when any one of the following criteria is met: i) the value of fitness function does not change in inner loop within \( N_N \) consecutive times; ii) the time of the new solution accepted reaches the given number \( N_I \); or; iii) the inner loop iteration reaches the given
number $N_s$.

### 3.2.5 SA algorithm

The detail steps of the improved SA for solving the DJTE are summarized as follows.

**Step 1** Initialize $T$, $\Delta t$, $T_s$, $N_s$, $N_I$, and $N_N$. Set $n_s = n_I = n_N = 0$. Set an initial solution $\lambda^0$ and $\beta^0$.

**Step 2** Obtain traffic pattern $\mathbf{f}^0$ through solving the DTA by the solution algorithm in Section 3.1 using the given values of $\lambda^0$ and $\beta^0$. Calculate objective function of the upper level $z^0 = z(\lambda^0, \beta^0, \mathbf{f}^0)$. Then, the fitness value of SA, $E^0 = E(\lambda^0, \beta^0, \mathbf{f}^0)$ according to Equation (26), could be obtained. Set $E_{best}^0 = E^0, \lambda_{best}^0 = \lambda^0, \beta_{best}^0 = \beta^0, \mathbf{f}_{best}^0 = \mathbf{f}^0$.

**Step 3** Generate new neighboring solutions $\lambda^1$ and $\beta^1$ by Equation (25).

**Step 4** Obtain traffic pattern $\mathbf{f}^1$ through solving the DTA by the solution algorithm in Section 3.1 using the given values of $\lambda^1$ and $\beta^1$. Calculate the fitness value of SA $E^1 = E(\lambda^1, \beta^1, \mathbf{f}^1)$.

**Step 5** Set $\Delta E = E^1 - E^0$. If $\Delta E \leq 0$ or $e^{-\Delta E/T} \geq random$, then accept the new solution as the current solution, update $\lambda^0 = \lambda^1, \beta^0 = \beta^1, \mathbf{f}^0 = \mathbf{f}^1$. If $E_{best}^1 \geq E^1$, then update $E_{best}^1 = E^1, \lambda_{best}^1 = \lambda^1, \beta_{best}^1 = \beta^1, \mathbf{f}_{best}^1 = \mathbf{f}^1$.

**Step 6** If $\left( (n_s \leq N_s) \text{ and } (n_I \leq N_I) \right) \text{ and } (n_N \leq N_N)$, then go to **Step 3**. Otherwise go to **Step 7**.

**Step 7** Set $T = T \times \Delta t$. If $T < T_s$, stop and output the best solution $\lambda_{best}^1, \beta_{best}^1, \mathbf{f}_{best}^1$ and the best fitness value $E_{best}$ of SA. Otherwise, go to **Step 3**.

### 4. NUMERICAL EXAMPLE

![Figure 1. A small test network](image)

To illustrate the proposed bi-level model formulation and solution algorithms, the small network shown in Figure 1 is adopted. This network consists of seven links, six nodes, and there are two simple paths for each OD pair (1,3) and (2,4). The attributes of the seven links
in this network are given in Table 2. In this example, it is assumed that the journey times on path 1, and traffic flow on link 2 can be observed.

<table>
<thead>
<tr>
<th>Link</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_a$ (min)</td>
<td>30</td>
<td>30</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>$C_a$ (pcu/min)</td>
<td>25</td>
<td>25</td>
<td>10</td>
<td>20</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$\omega_a$</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Due to the symmetric network structure and input data, path 1 and 4 (path 2 and 3) will have exactly the same traffic conditions. Hence, in this numerical example, only the numerical results on paths 1 and 2 will be investigated. The time of interest in this numerical example is the morning period (6:00AM-10:00AM), while the demand departure time interval is taken as 1 minute. The weighting parameters in objective function are set as $\gamma_1 = \gamma_2 = 0.5$. The parameter in fitness function is set as $\rho = 0.5$ and the parameter for link travel time covariance in Equation (7) is set as $\theta = 0.5$. The confidence level of path journey time reliability, $\alpha$, is assumed to be 90% in the lower-level DTA problem. The prior traffic demands of each OD pair at the beginning of time interval $k$ is assumed to be calculated through Equation (28)

$$d^{13}(t) = d^{24}(t) = 46.2 \times \exp(-0.006t - 0.8)^2$$ (28)

Assuming the demand multiplier for the actual demand are: $\lambda^* = [1, 1]$, and the multiplier between the mean and variance of the link travel time $\beta^* = 0.25$. With the actual traffic demands defined in Equation (28) and the demand multiplier $\lambda^*$, the observed data (path journey times and link flows) is generated by solving the lower-level DTA model. For algorithm to solve the DTA, the stopping tolerance $\varepsilon$ is set to be 0.00001, the maximum iteration number $N$ is set to be 1000.

For the upper level model, the upper and lower bound of demand multiplier $\lambda$ adopted in constraint (21) is respectively taken as $\lambda^- = [0.5, 0.5]$ and $\lambda^+ = [1.5, 1.5]$. According to the literatures and some tests which have been carried out to determine the appropriate parameters for the improved SA approach, the following values are adopted for the SA parameters in this numerical example: $M = 1000$, $T = 100$, $\Delta t = 0.9$, $T_s = 1$, $N_s = 200$, $N_i = 100$, $N_N = 50$.

To quantify the accuracy of DJTE estimation results, the following Root Mean Squared Errors (RMSEs) are adopted as the performance measures.

$$RMSE_{\mu} = \sqrt{\frac{1}{N_{\text{path}} N_{T_s}} \sum_{\text{p=1}}^{\text{p} \in \text{p}_s} \sum_{k=k_{T_s}}^{k_{T_s}} (\mu_p^*(k) - \overline{\mu}_p(k))^2}$$

$$RMSE_{\sigma} = \sqrt{\frac{1}{N_{\text{path}} N_{T_s}} \sum_{\text{p=1}}^{\text{p} \in \text{p}_s} \sum_{k=k_{T_s}}^{k_{T_s}} (\sigma_p^*(k) - \overline{\sigma}_p(k))^2}$$
\[ RMSE_\lambda = \sqrt{\frac{1}{N_{OD}} \sum_{r \in R} (\lambda_{rs} - \hat{\lambda}_{rs})^2} \]

\[ RMSE_\beta = |\beta - \hat{\beta}| \]

where \( N_{path} \) is the total number of paths in the network; \( N_{T_i} \) is the total number of time intervals; \( N_{A} \) is the total number of links in the network; \( N_{OD} \) is the total number of OD pairs in the network. With the above definitions, the proposed DJTE model is solved and the results are shown in Table 3.

<table>
<thead>
<tr>
<th>Observed value</th>
<th>Estimated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>[1 1]</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.25</td>
</tr>
<tr>
<td>RMSE( \mu )</td>
<td>0.1374</td>
</tr>
<tr>
<td>RMSE( \sigma )</td>
<td>0.0690</td>
</tr>
<tr>
<td>RMSE( \lambda )</td>
<td>0.0748</td>
</tr>
<tr>
<td>RMSE( \beta )</td>
<td>0.0017</td>
</tr>
</tbody>
</table>

In Table 3, it could be seen that the estimated \( \lambda \) (0.9091 and 1.0542) and \( \beta \) (0.2517) is relatively close to the observed values. By comparing the RMSE\( \mu \) and RMSE\( \sigma \) with the range of mean travel time (28 minutes ~ 38 minutes in Figure 2) and standard deviation (SD) of travel time (7.5 minutes ~ 12 minutes in Figure 3), it could be concluded that in this numerical example, the proposed model provides a relatively good estimation of the mean and SD, or variance, of travel time. Considering RMSEs of the parameters, it could be seen that the model give a more accurate estimation for the multiplier between mean and variance of link travel times (\( \beta \)). In order to have a more detail analysis on the temporal performance of the proposed model in the estimation of path journey times, the observed and estimated mean path journey times (Figure 2), SD of path journey times (Figure 3), effective path journey times (Figure 4) and path inflows (Figure 5) for path 1 and 2 are plotted and discussed.

![Figure 2. Comparison between observed and estimated mean of path journey times](image)
Comparing the difference of the observed and estimated mean journey time of the two paths in Figure 2, it could be seen that the DJTE model proposed in this paper perform better in the early morning (from 6:00AM to 7:00AM) than in the peak hour period (from 8:00AM-9:00AM). Such difference of performance could be explained by the increasing variability of demands and the highly dynamics traffic flow propagation processes in the peak hour period (i.e., 8:00AM-9:00AM). Apart from the accuracy of estimation, it could also be observed that the proposed DJTE model generally underestimates the mean path journey time for all departure time intervals.

Figure 3. Comparison between observed and estimated SD of path journey times

From the observed and estimated SDs of journey time for the two paths, despite there is some overestimations at some particular time intervals (e.g., path 1 at around 7:15AM), it could be seen that the DJTE model proposed in that paper give a relatively good estimation of the SD of path journey times.

If travelers only consider mean path journey time in making their route choice, based on the results shown in Figure 2, travelers will choose path 2 only as the mean path journey times of path 2 are smaller than that of path 1. However, there are positive inflows for both of the paths (Figure 5). This could be explained by the fact that in this numerical example, the variance of path journey time of path 2 is larger than that of path 1 (Figure 3). Thus, with the consideration of the variance of path journey time, the overall path journey time of path 2 may exceed path 1. Therefore, in the lower level model of the proposed DJTE, the reliability of path journey time is incorporated into the travelers’ route choice in solving the DTA (Equation 6). In this numerical example, with the assumption of 90% chance of arriving the destination within the path travel time budget (i.e., $\alpha = 0.9$), the observed and estimated effective path journey time for path 1 and 2 are shown in Figure 4.

In Figure 4, it could be seen that the effective path journey time for path 1 and 2 are exactly the same for all departure time as both of the paths are used (Figure 5). The variation of the effective travel time (Figure 4) is similar to that for the mean path journey time (Figure 2) with increasing trend toward the peak period (8:00AM ~ 9:00AM). Similarly, the proposed DJTE model has a larger estimation error in the peak period and in general underestimating the effective path journey time.
As shown in Figure 4, it is clearly illustrated that our DJTE can estimate the effective path journey time for travelers whenever he (she) departs. It reveals that when the traffic condition become more congested (demand increasing), the effective path journey time will be greater. It can be conclude that confronted with the increasing path journey time, travelers should depart as earlier as possible to avoid the peak hour period.

![Figure 4: Comparison between observed and estimated effective path journey times](image1)

**Figure 4. Comparison between observed and estimated effective path journey times**

Figure 5 shows the observed and estimated path inflows of the two paths. It could be seen that the estimated errors for path 2, which has a larger inflow, are greater than that for path 1. With the estimated demand multipliers are equal to [0.9091 1.0542], the demand for OD pair (1,3) is underestimated, while that for OD pair (2,4) is overestimated. As paths 2 and 3 have common link 4, travel costs on these two paths are highly correlated. The overestimated demand multiplier for OD pair (2,4) will increase the travel cost on path 3, as a result, the travel cost on path 2 will also increase. Thus, the path inflow on path 2 will surely be underestimated.

![Figure 5: Comparison between observed and estimated path inflows](image2)

**Figure 5. Comparison between observed and estimated path inflows**

5. CONCLUSIONS
In this paper, a bi-level programming model for dynamic journey time estimation (DJTE) is proposed. A simulated annealing (SA) solution algorithm was modified with the potential global search ability to solve the DJTE problem. In the proposed model, the objective of the upper level problem is a weighed least squared function to minimize the difference of mean and SD between the observed and estimated path travel times. In the low level problem, the reliability-based dynamic traffic assignment model is employed to account for travelers’ path choice behaviors under supply uncertainty. A route-based or path-based approach with the use of the MSA algorithm is applied to solve the lower level DTA problem, by which the traffic pattern (e.g. path flows, link flows, path travel times, link travel times) could be obtained. Numerical examples are used to illustrate the application of the proposed DJTE model and the modified SA algorithm.

This paper proposed a modified SA algorithm for solving the DJTE problem which is a heuristic in nature. One of our future research directions is to investigate the potential for using other more efficient algorithms for solving the DJTE problem in realistic networks. Future works also include the consideration of demand uncertainty in road network and to incorporate other types of link travel time functions into the reliability-based DTA model.

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APPENDIX A - COMPUTATION OF MEAN AND SD OF THE PATH JOURNEY TIMES

Assume a path $p$ between OD pair $(r,s)$ is consist of the following sets of links \{1,2,$\ldots$,m\} and nodes $(r,1,2,\ldots,m-1,s)$. Obviously,

\[ t_p^r(k) = t_{a_1}(k) \]
\[ \sigma_p^r(k) = \sigma_{a_1}(k) \]
\[ t_p^s(t) = t_{a_1}(t) + t_{a_2}(t + t_{a_1}(t)) \]
\[ (\sigma_p^s(t))^2 = (\sigma_{a_1}(t))^2 + (\sigma_{a_2}(t + t_{a_1}(t)))^2 + 2\text{cov}(t_{a_1}(t),t_{a_2}(t + t_{a_1}(t))) \]
\[ \ldots \]

Then we can express the mean of path travel time on path $p \in P_{rs}$ of pair $(r,s)$ at the end of time interval $k \in T_d$, $t_p^{rs}(t)$, by recursively applying the mean of link travel time.

\[ t_p^{rs}(t) = t_{a_1}(t) + t_{a_2}(t + t_{a_1}(t)) + \ldots + t_{a_n}(t + t_{a_{n-1}}(t)) + t_{a_{n-1}}(t + t_{a_{n-2}}(t + \ldots + t_{a_1}(t + t_{a_2}(t + \ldots + t_{a_1}(t))))) \]
\[ = t_{a_1} + t_{a_2}(t+t_{a_1}) + \ldots + t_{a_n}(t+t_{a_{n-1}}) \]
\[ (\sigma_p^{rs}(t))^2 = (\sigma_{a_1}(t))^2 + (\sigma_{a_2}(t + t_{a_1}(t)))^2 + \ldots + (\sigma_{a_n}(t + t_{a_{n-1}}(t + t_{a_{n-2}}(t + \ldots + t_{a_1}(t))))^2 \]
\[ + 2 \sum_{1 \leq l \leq m} \text{cov}(t_{a_1}(t+t_{a_1}(t) + \ldots + t_{a_{n-1}}(t + \ldots + t_{a_1}(t)) + t_{a_2}(t + \ldots + t_{a_{n-1}}(t)), t_{a_2}(t + \ldots + t_{a_{n-1}}(t) + \ldots + t_{a_1}(t))) \]
\[ = (\sigma_{a_1}(t))^2 + (\sigma_{a_2}(t + t_{a_1}(t)))^2 + \ldots + (\sigma_{a_n}(t + t_{a_{n-1}} + \ldots + t_{a_1}(t)))^2 \]
\[ + 2 \sum_{1 \leq l \leq m} \text{cov}(t_{a_1},t_{a_2}) \]

where $t_{a_1} = t_{a_1}(t), t_{a_2} = t_{a_2}(t + t_{a_1}(t)), \ldots$ for short. We can rewritten Equation (A1) and Equation (A2) as

\[ t_p^{rs}(t) = \sum_{a \text{ on path } p} \sum_{l \in \mathbb{K}} t_{a}(l)\delta^{rs}_{apk}(l) \quad (A3) \]
\[ (\sigma_p^{rs}(t))^2 = (\sum_{a \text{ on path } p} \sum_{l \in \mathbb{K}} \sigma_{a}(l)\delta^{rs}_{apk}(l))^2 + 2 \sum_{1 \leq l \leq m} \text{cov}(t_{a_1},t_{a_2}) \quad (A4) \]

where $\delta^{rs}_{apk}(l)$ is equal to 1, if the flow on path $p$ of OD pair $(r,s)$ entering the network at interval $k$ and arrive link $a$ at interval $l$; otherwise,0. The detail information as follows:

\[ \delta^{rs}_{apk}(l) = \begin{cases} 1 & \text{If } k + t_{a_1} + t_{a_2} + \ldots + t_{a_{n-1}} = l \\ 0 & \text{Otherwise.} \end{cases} \]

And for any link $a$ on path $p$, clearly

\[ \sum_{l \in \mathbb{K} \cap T_d} \delta^{rs}_{apk}(l) = 1 \quad \forall p \in P_{rs}, rs \in R, k \in T_d \]
REFERENCES


